Homework 4

Due: Thursday May 2, 2013

1. Show that if there exists a code with word length N, M words and minimum distance d with d even, then there exists a code with word length N, M words, and minimum distance d such that every word has an even number of 1's.

2. What is the largest possible number of words in a code with length 6 and minimum distance 3?

3. Let C be a binary code of length n and minimum distance d. Define a new code C' of length $n+1$ by

$$C' = \{x_1 x_2 \dots x_{n+1} | x_1 x_2 \dots x_n \in C,\ x_1 + \dots + x_{n+1} \equiv 0 \pmod 2\}$$

   Suppose that d is odd. Prove that the minimum distance of C' is $d+1$.

4. Let C be the binary linear code with generator matrix

$$G = \begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 & 1 \end{pmatrix}$$

   (a) Give a complete list of all the codewords of C.

   (b) What is the minimum distance of C? How many errors can it correct?

   (c) Decode the words 1100111, 0010110.

5. Consider the binary code with four codewords $\{(0,0,1),(1,1,1),(1,0,0),(0,1,0)\}$. Show that this code is not linear and compute its minimum distance.

6. Suppose $k < n + 1 - \log_2(\sum_{i=0}^{d-1} \binom{n}{i})$. Prove that there exists a linear $[n,k,d]$ code.

7. Let k be an integer. There are $2^k - 1$ prisoners. A black or white hat is to be placed on the head of each prisoner. Each prisoner will be able to see the colour of the hats worn by every other prisoner, but not the colour of his or her own hat. From this point in time, no communication between the prisoners is possible. Each prisoner is then simultaneously asked: "What colour is your hat?" There are three allowed responses, white, black and no response. If at least one person guesses their hat colour correctly and noone guesses the wrong colour, the prisoners are set free. Otherwise they are all executed.

   The prisoners may meet beforehand to decide on their strategy. What is their best course of action? Assume that their common goal is to be freed.