Homework 2

Due: Thursday April 18, 2013

1. Let $z : [a, b] \to \mathbb{C}$ be differentiable at $c \in (a, b)$ and let $F : \Omega \to \mathbb{C}$ be holomorphic, where Ω is an open set containing $z(c)$. Prove that the composition $F \circ z$ is differentiable at c with
$$(F \circ z)'(c) = F'(z(c))z'(c).$$

2. Determine the radius of convergence of the series $\sum_{n=1}^{\infty} a_n z^n$ when

   (a) $a_n = (\log n)^2$

   (b) $a_n = n!$

   (c) $a_n = \frac{n^2}{3^n+2n}$

   (d) $a_n = (n!)^3/(3n)!$ (Stirling's formula may be useful here)

3. Compute the line integral
$$\int_C \frac{z^2}{z-1} dz$$
where C describes the circle with centre 0 and radius 3 traversed anticlockwise.

4. Let $F : \mathbb{C} \to \mathbb{C}$ be a holomorphic function (holomorphic functions with domain $\mathbb{C}$ are called *entire*) and let P be a polynomial with real coefficients. Suppose that for all $z \in \mathbb{C}$ we have
$$|F(z)| < P(|z|).$$
Prove that F is a polynomial of degree at most the degree of P. (when P has degree zero this is called Liouville's Theroem).