

Homework 5

Due: Thursday May 16, 2013

1. Compute $\Gamma'(1)$.

2. Prove the formula

$$\sin(\pi z) = \pi z \prod_{n=1}^{\infty} \left(1 - \frac{z^2}{n^2}\right)$$

(a) without using Hadamard's factorization theorem

(b) using Hadamard's factorization theorem (the statement is in the textbook)

3. Prove that

$$\zeta(s)^2 = \sum_{n=1}^{\infty} \frac{d(n)}{n^s}$$

where $d(n)$ is the number of divisors of n .

4. Prove that

$$\zeta(s) = \prod_p (1 - p^{-s})^{-1}$$

for $\Re(s) > 1$ where the product is over all primes. Use this to show that $\zeta(-3+47i) \neq 0$.