Homework 5

Due: Thursday May 24, 2012

1. Using the definition of a nilpotent group as given in class in terms of the lower central series, prove that a p-group is nilpotent.

2. Let F be a field. Prove that the group of upper-triangular matrices with entries in F is solvable.

3. Find a Sylow 3-subgroup of S_9.

4. Prove that if a and b are elements of a ring, then $(-a)b = -(ab)$.

5. An element x of a ring is called *nilpotent* if some power of x is zero. Prove that if x is nilpotent, then $1 + x$ is a unit.

6. Let $f(x)$ and $g(x)$ be polynomials with coefficients in a ring R and with $f \neq 0$. Prove that if the product $f(x)g(x)$ is zero, then there exists a nonzero element c of R with $cg(x) = 0$.