

## COMMENTS, ERRATA, ETC.

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ABSTRACT. This is a compilation of my own comments on my papers.

### [1] FACTORIAL GROTHENDIECK POLYNOMIALS

This paper is essentially my honours thesis. Factorial Grothendieck polynomials represent the classes in equivariant K-theory of Schubert varieties in the Grassmannian. This is not discussed in this paper, but follows from the comparison made with double Grothendieck polynomials once one knows how the double Grothendieck polynomials represent classes in the equivariant K-theory of the flag variety. For a better relationship between the factorial and double Grothendieck polynomials, see <http://petermc.net/maths/papers/fgpaddendum.pdf>.

### [3] METAPLECTIC WHITTAKER FUNCTIONS AND CRYSTAL BASES

This, together with [5], is essentially my PhD thesis.

### [4] FACTORIAL SCHUR FUNCTIONS VIA THE SIX VERTEX MODEL

This was never published because [7] contains strictly stronger results.

### [5] PRINCIPAL SERIES REPRESENTATIONS OF METAPLECTIC GROUPS OVER LOCAL FIELDS

Proof of Theorem 4.2: The fact that the index of  $\mathbf{M}(k)$  in  $\mathbf{G}(k)$  is coprime to  $n$  is stated without proof. The case where  $\mathbf{G} = GL_N$  essentially appears as problem A4 in the 2022 Simon Marais competition and the solution in [Mc] shows how to prove this result in the generality needed for this paper.

Proof of Theorem 12.2, 3rd line:  $p(t) \in T \cap K$  should be replaced by  $p(tkt^{-1}) \in T \cap K$ .

Proof of Theorem 12.3: The “appropriate volume” is not  $q^{2n_\alpha-1}$ , it is  $q^l - q^{l-1}$ . To compute this volume, we have to know when  $h = \begin{pmatrix} b\varpi^{2l} & d \\ -a & -u\varpi^{-l} \end{pmatrix}$  and  $H = \begin{pmatrix} B\varpi^{2l} & D \\ -A & -U\varpi^{-l} \end{pmatrix}$  are in the same left  $I$ -coset. This happens precisely when  $h^{-1}H \in I$ , which upon multiplying out leads to the condition  $(-Du + Ud)\varpi^{-l} \in \mathcal{O}_F$ . Since  $d, u, D, U$  are all units, this is equivalent to  $D/U \cong d/u \pmod{\varpi^l}$  and there are  $q^l - q^{l-1}$  units in  $\mathcal{O}_F/\varpi^l$ , hence  $q^l - q^{l-1}$  cosets, hence this is the correct volume.

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[8] FINITE DIMENSIONAL REPRESENTATIONS OF KHOVANOV-LAUDA-ROUQUIER  
ALGEBRAS I: FINITE TYPE

A small missing argument in types ADE in positive characteristic is completed in [9]. Alternative arguments that avoid a case-by-case analysis in type ADE appear in [10] (I have not checked if they generalise to types BCFG - the key argument is the one using  $R$ -matrices in [10, §15]).

The homological properties proved in this paper essentially imply that finite type KLR algebras are affine quasi-hereditary, see [9] where the theory of standard modules is developed, and [14, §6], which collates the results.

[9] HOMOLOGICAL PROPERTIES OF FINITE TYPE KHOVANOV-LAUDA-ROUQUIER  
ALGEBRAS (WITH J. BRUNDAN AND A. KLESHCHEV)

The results in this paper can be restated in the language of affine quasi-hereditary algebras introduced in [K], and a discussion of these results in this language is in [14, §6].

[10] REPRESENTATIONS OF KHOVANOV-LAUDA-ROUQUIER ALGEBRAS III: SYMMETRIC  
AFFINE TYPE

The strange numbering of this paper (there is no II) is because originally II was going to be a paper about affine  $\mathfrak{sl}_2$ , but the results in this paper were obtained before the other was written up, and supercede them.

[13] FOLDING KLR ALGEBRAS

This paper was written in order to provide the foundations for [19]. The identification of the ring  $A$  with  $\mathbb{Z}$  in all types is obtained in [MSZ].

[18] SINGULARITIES OF SCHUBERT VARIETIES WITHIN A RIGHT CELL (WITH M. LANINI)

The discussion in §4.3 can be simplified using [17, Theorem 5.1] which gives an alternative and much more general family of sheaves, which agrees with parity sheaves on Schubert varieties, and whose theory is more amenable to restriction along a slice.

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