

The book of Ohtsuki has been recommended to me as the most readable source amongst the books listed as references.

1 Introduction

Overview of what we hope to achieve this semester

2 Topology

What is a knot? A link? The Reidemeister moves. Presenting a knot as the closure of a braid. Markov moves. Depending on time, some classical knot invariants (in order of difficulty: tricolourability, fundamental group of knot complement, Alexander polynomial).

3 The Jones polynomial

The Jones polynomial via Skein Theory and Kauffman bracket. The Jones polynomial via an R-matrix.

4 Ribbon Categories

Graphical calculus for tensor categories and duals. What is a ribbon category and how can it be used to produce an invariant of a knot.

5 Ribbon Hopf algebra

How a ribbon Hopf algebra can be used to produce a ribbon category. Example of $U_q(\mathfrak{sl}_2)$ as explicitly as possible.

6 Quantum groups

What $U_q(\mathfrak{g})$ is and its ribbon Hopf structure.

7 Jones again

A review of what we've covered. Extracting the Jones polynomial from the quantum group perspective. Quantum origin of the skein relations. Why do we need the ribbon element.

8 HOMFLY polynomial

Uses the presentation of a knot as the closure of a braid. Jones-Ocneanu trace on the Hecke algebra.

9 Further topics

Extra fun more advanced material. Quantum invariants of 3-manifolds. Khovanov homology.