MATH 116 Midterm. Tue May 7 2013.

Instructions

- Put your name on your answer booklet and sign the Honor Code statement.
- Begin each question on a new page.
- Give complete proofs to all questions, unless asked otherwise.
- You may use any result proved in class or in the homeworks, unless the question explicitly asks you to prove such a result.
- You may take in two sheets of letter or A4 sized paper with notes on it (double sided allowed). No additional notes, books, computers or other outside help is allowed.
- The exam lasts 75 minutes.
- Questions may be submitted to Peter McNamara in 382H.

1. [3 points each] Determine whether the following functions have poles at $z = 0$ and if so, determine their residues.
   (a) $z^{-2}(z+2)^3$
   (b) $\Gamma(z)$
   (c) $\sin(1/z)$.
   (d) $\zeta(z)$.
2. [12 points] Let $f : \Omega \to \mathbb{C}$ be an analytic function and $z_0 \in \Omega$ a point with $f(z_0) = 0$. Suppose f is not the zero function. Give a proof that there exists a disc centred at z_0 for which the only zero of f in this disc is at z_0.
3. [12 points] Prove that the formula
$$f(z) = \sum_{n \in \mathbb{Z}} \frac{1}{(z+n)^2}$$
defines a meromorphic function on $\mathbb{C}$.
4. [12 points] Define
$$\sum_{n=0}^{\infty} a_n z^n = \frac{e^{-z-z^2/2}}{1-z}.$$
Find an asymptotic formula for a_n.