More Geometry Questions

(also in no particular order)

1. Let ABC be a triangle, $D \in (BC)$ such that $AD \perp BC$ and let E, F be the centres of the circles inscribed into triangles ABD and ACD respectively. Line EF intersects AB and AC in K and L, respectively. Show that $AK = AL$ if and only if $AB = AC$ or $\angle A = 90^\circ$.

2. Let $ABCD$ be a convex quadrilateral. On AB, BC, CD, DA, points M, N, P, Q are chosen such that $AQ = DP = CN = BM$.
Show that if $MNPQ$ is a square, then $ABCD$ is also a square.

3. Let triangle ABC have orthocentre H, and let P be a point on its circumcircle. Let E be the foot of the altitude BH, ~~let~~ let $PAQB$ and $PARC$ be parallelograms and let AQ meet HR in X. Prove that $EX \parallel AP$.

4. Let P be a point inside $\triangle ABC$ such that $\angle APB - \angle C = \angle APC - \angle B$. Let D, E be incentres of $\triangle APB$, $\triangle APC$ respectively. Show that AP, BD and CE meet in a point.

5. Let ABC be an acute-angled triangle with $BC > CA$. Let O be its circumcentre, H its orthocentre and F the foot of its altitude CH. Let the perpendicular to OF at F meet the side CA at P. Prove that $\angle FHP = \angle BAC$.

6. Let A_1 be the centre of the square inscribed in acute triangle ABC with two vertices of the square on BC and the others on AB and AC. B_1 and C_1 are defined similarly. Prove that AA_1, BB_1, CC_1 are concurrent.

7. Let A, B, C, A', B', C' be points on a circle such that $AA' \perp BC$, $BB' \perp CA$, $CC' \perp AB$. Further, let D be a point on that circle and let DA' intersect BC in A'', DB' intersect CA in B'', DC' intersect AB in C'' all line segments being extended when required. Prove that A'', B'', C'' and the orthocentre of $\triangle ABC$ are collinear.

8. Let ABC be a triangle and D the foot of the altitude from A. Let E and F be on a line passing through D such that $AE \perp BE$, $AF \perp CF$, and E and F are different from D. Let M and N be midpoints of the line segments BC and EF respectively. Prove that $AN \perp NM$.

9. A parallelogram $ABCD$ is given in the plane. A circle with a centre P touches its side BC and the continuations of the side AB and ~~AC~~ the diagonal AC. A circle with centre Q touches the side CD and the continuations of the side AD and the diagonal AC. Let K be the point of tangency of the first circle with the line AB and L be the point of tangency of the second circle with the line AD. Let also M be the point of intersection of AB and QC, and N the point of intersection of AD and PC. Prove that $KM = NL$