COMMENTS, ERRATA, ETC.

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Abstract. This is a compilation of my own comments on my papers.

[1] Factorial Grothendieck Polynomials

This paper is essentially my honours thesis. Factorial Grothendieck polynomials represent the classes in equivariant K-theory of Schubert varieties in the Grassmannian. This is not discussed in this paper, but follows from the comparison made with double Grothendieck polynomials once one knows how the double Grothdieck polynomials represent classes in the equivariant K-theory of the flag variety. For a better relationship between the factorial and double Grothendieck polynomials, see `http://petermc.net/maths/papers/fgpaddendum.pdf`.

[3] Metaplectic Whittaker Functions and Crystal Bases

This, together with [5], is essentially my PhD thesis.

Remark 8.5: The conjecture made here is false. A counterexample in G_2 appears in [L].

[4] Factorial Schur Functions via the Six Vertex Model

This was never published because [7] contains stricly stronger results.

[5] Principal Series Representations of Metaplectic Groups Over Local Fields

p5: The remark (that isn't used in the paper) that [AdCK] produces a central extension whose commutator is the Hilbert symbol is not correct, the commutator is only the Hilbert symbol up to a sign.

Proof of Theorem 4.2: The fact that the index of $\mathbf{M}(k)$ in $\mathbf{G}(k)$ is coprime to n is stated without proof. The case where $\mathbf{G} = GL_N$ essentially appears as problem A4 in the 2022 Simon Marais competition and the solution in [Mc] shows how to prove this result in the generality needed for this paper.

Proof of Theorem 12.2, 3rd line: $p(t) \in T \cap K$ should be replaced by $p(tkt^{-1}) \in T \cap K$.

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Theorem 12.3: The condition for relation (4) should be $\langle \alpha^\vee, \lambda \rangle = 2n_\alpha$ (we apologise for using α for a coroot instead of a root).

Proof of Theorem 12.3: The "appropriate volume" is not $q^{2n_\alpha - 1}$, it is $q^l - q^{l-1}$. To compute this volume, we have to know when $h = \begin{pmatrix} b\varpi^{2l} & d \\ -a & -u\varpi^{-l} \end{pmatrix}$ and $H = \begin{pmatrix} B\varpi^{2l} & D \\ -A & -U\varpi^{-l} \end{pmatrix}$ are in the same left I-coset. This happens precisely when $h^{-1}H \in I$, which upon multiplying out leads to the condition $(-Du + Ud)\varpi^{-l} \in \mathcal{O}_F$. Since d, u, D, U are all units, this is equivalent to $D/U \cong d/u \pmod{\varpi^l}$ and there are $q^l - q^{l-1}$ units in $\mathcal{O}_F/\varpi^l$, hence $q^l - q^{l-1}$ cosets, hence this is the correct volume. The computation/proof discussed here should really have been done by first applying an Iwasawa factorisation to $\left(\begin{smallmatrix} a & b \\ c & d \end{smallmatrix}\right)$ to make it simpler.

[8] Finite Dimensional Representations of Khovanov-Lauda-Rouquier Algebras I: Finite Type

A small missing argument in types ADE in positive characteristic is completed in [9]. Alternative arguments that avoid a case-by-case analysis in type ADE appear in [10] (I have not checked if they generalise to types BCFG - the key argument is the one using R-matrices in [10, §15]).

The homological properties proved in this paper essentially imply that finite type KLR alegbras are affine quasi-hereditary, see [9] where the theory of standard modules is developed, and [14, §6], which collates the results.

[9] Homological Properties of Finite Type Khovanov-Lauda-Rouquier Algebras (with J. Brundan and A. Kleshchev)

The results in this paper can be restated in the language of affine quasi-hereditary algebras introduced in [K], and a discussion of these results in this language is in [14, §6].

[10] Representations of Khovanov-Lauda-Rouquier algebras III: Symmetric affine type

The strange numbering of this paper (there is no II) is because originally II was going to be a paper about affine $\mathfrak{sl}_2$, but the results in this paper were obtained before the other was written up, and supercede them.

Proposition 9.2: $R(n\delta)$ should be $R(n\alpha)$.

[13] Folding KLR algebras

This paper was written in order to provide the foundations for [19]. The identification of the ring A with $\mathbb{Z}$ in all types is obtained in [MSZ].

[18] Singularities of Schubert varieties within a right cell (with M. Lanini)

The discussion in §4.3 can be simplified using [17, Theorem 5.1] which gives an alternative and much more general family of sheaves, which agrees with parity sheaves on Schubert varieties, and whose theory is more amenable to restriction along a slice.

References

[AdCK] E. Arbarello, C. de Concini and V. G. Kac. The infinite wedge representation and the reciprocity law for algebraic curves. (1989)

[K] Alexander S. Kleshchev. Affine highest weight categories and affine quasihereditary algebras. *Proc. Lond. Math. Soc. (3)*, 110(4):841–882, 2015. arXiv:1405.3328

[L] Spencer Leslie, Resonant Mirkovic-Vilonen polytopes and formulas for highest-weight characters. *Selecta Mathematica New Ser.* 2019 arXiv:1808.10508

[MSZ] Ying Ma, Toshiaki Shoji and Zhiping Zhou. Foldings of KLR algebras. arXiv:2210.10279

[1] Peter J. McNamara Factorial Grothendieck polynomials. *Electron. J. Combin.*, 13(1):Research Paper 71, 40 pp. (electronic), 2006.

[3] Peter J. McNamara. Metaplectic Whittaker Functions and Crystal Bases. *Duke Math J.* 156 (2011), no 1, 1–31.

[4] Peter J. McNamara. Factorial Schur Functions via the Six Vertex Model arXiv:0910.5288

[5] Peter J. McNamara. Principal Series Representations of Metaplectic Groups Over Local Fields. Multiple Dirichlet Series, L-functions and Automorphic Forms, Birkhauser Progress in Math. 300 (2012) 299–327. arXiv:0911.2274

[7] D. Bump, P. J. McNamara, and M. Nakasuji. Factorial Schur Functions and the Yang-Baxter Equation. *Comment. Math. Univ. St. Pauli* 63 (2014), no. 1-2, 23–45 arXiv:1108.3087

[8] Peter J. McNamara. Finite Dimensional Representations of Khovanov-Lauda-Rouquier Algebras I: Finite Type. *J. Reine Angew. Math.* 707 (2015), 103–124.

[9] J. Brundan, A. Kleshchev and P. J. McNamara, Homological Properties of Finite Type Khovanov-Lauda-Rouquier Algebras.

[10] Peter J. McNamara. Representations of Khovanov-Lauda-Rouquier algebras III: Symmetric affine type. *Math. Z.* 287:(1-2):243–286, 2017. arXiv:1407.7304.

[13] Peter J. McNamara. Folding KLR algebras. *J. Lond. Math. Soc. (2)*, 100(2):447–469, 2019. arXiv:1603.05768.

[14] Peter J. McNamara Representation Theory of Geometric Extension Algebras. arXiv:1701.07949

[17] Peter J. McNamara Non-perverse parity sheaves on the flag variety. arXiv:1812.00178

[18] M. Lanini and P. J. McNamara. Singularities of Schubert varieties within a right cell. *SIGMA* 17 (2021), 070, 9pp. arXiv:2003.08616

[19] Peter J. McNamara. Cluster Monomials are Dual Canonical. arXiv:2112.04109.

[Mc] Peter J. McNamara. SMCC 2022 A4. http://www.petermc.net/blog/2022/10/20/smmc-2022-a4/, 2022.

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