Addendum to Factorial Grothendieck Polynomials

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Abstract

In [Mc], the relationship between factorial Grothendieck and double Grothendieck polynomials is only discussed in the limit as the number of variables tends to infinity. In fact this limiting process is unnecessary, and the purpose of this note is to provide the more precise result. All notation and references are from [Mc]. As a special case, this includes the statement that factorial Schur polynomials are double Schubert polynomials for Grassmannian permutations.

Let λ be a parition. Let $p \geq \ell(\lambda)$ be an integer. Let $n \geq p + \lambda_1$. Associated to λ is the permutation $w(\lambda) \in S_{n+1}$ as defined in Section 8.

Theorem 0.1 (Relationship between factorial and double Grothendieck polynomials)**.** *The factorial Grothendieck polynomial in p variables is a double Grothendieck polynomial,*

$$\mathcal{G}_{w(\lambda)}(x;y) = G_\lambda(x_1, \ldots, x_p|y).$$

Proof. We know from Lemma 7.7 and Theorem 7.8 that $\mathcal{G}_{w(\lambda)}$ is the coefficient of $u_{w(\lambda)}$ in the product

$$\prod_{i=1}^{n} \prod_{j=n+1-i}^{1} h_{i+j-1}(x_i \oplus y_j).$$

By the last statement in Lemma 8.2, $\mathcal{G}_{w(\lambda)}$ is the coefficient of $u_{w(\lambda)}$ in the product

$$\prod_{i=1}^{p} \prod_{j=n+1-i}^{1} h_{i+j-1}(x_i \oplus y_j).$$

Now $\mathcal{G}_{w(\lambda)}$ is in the image of the divided difference operator ∂_i for $1 \leq i < p$. Here

$$\partial_i f = \frac{f(\ldots, x_i, x_{i+1}, \ldots) - f(\ldots, x_{i+1}, x_i, \ldots)}{x_i - x_{i+1}}.$$

The divided difference operator squares to zero, so $\mathcal{G}_{w(\lambda)}$ is also in its kernel, hence $\mathcal{G}_{w(\lambda)}$ is symmetric in $x_1, \ldots, x_p$. Thus $\mathcal{G}_{w(\lambda)}$ is the coefficient of $u_{w(\lambda)}$ in the product

$$\prod_{i=1}^{p} \prod_{j=n+1-i}^{1} h_{i+j-1}(x_{p+1-i} \oplus y_j).$$

Now we argue as in the proof of Theorem 8.6 to prove our theorem. $\square$

References

[Mc] Peter J. McNamara. Factorial Grothendieck polynomials. *Electron. J. Combin.*, 13(1):Research Paper 71, 40 pp. (electronic), 2006.